Design and Analysis of Multi-User SDMA Systems with Noisy Limited CSIT Feedback

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Abstract*—In this paper, we consider spatial-division multiple-access (SDMA) systems with one base station with multiple antennae and a number of single antenna mobiles under noisy limited CSIT feedback. We propose a robust noisy limited feedback design for SDMA systems. The solution consists of a real-time robust SDMA precoding, user selection and rate adaptation as well as an offline feedback index assignment algorithm. The index assignment problem is cast into a Traveling Sales Man problem (TSP). Based on the specific structure of the feedback constellation and the precoder, we derive a low complex but asymptotically optimal solution. Simulation results show that the proposed framework has significant goodput gain compared to the traditional naive designs under noisy limited feedback channel. Furthermore, we show that the average system goodput scales as $\mathcal{O}(\frac{n_T(1-\epsilon)}{n_T-1}(C_{fb}-\log_2(N_n)))$ and $\mathcal{O}(n_T\cdot\log_2 P)$ in the interference limited regime ($C_{fb}<(n_T-1)\log_2 P+\log_2 N_n$) and noise-limited regime respectively. Hence, despite the noisy feedback channel, the average SDMA system goodput grows with the number of feedback bits in the interference limited regime while in noise limited regime increases linearly with the number of transmit antenna and the forward channel SNR ($\log_2 P$).

I. INTRODUCTION

It is widely known that spatial-division multiple-access(SDMA) is an important technique to enhance the throughput of multi-user wireless systems due to spatial multiplexing. However, SDMA system requires channel state information at transmitter(CSIT). In FDD systems, only a limited number of bits(e.g. 6 bits for WiMAX [3]) can be allocated to carry the CSIT feedback, namely the *limited feedback*. In [1], the authors consider multiuser MISO system with a limited total feedback bits constraint and proposed a codebook design algorithm and a CSIT decomposition algorithm. The authors of [2] analyzed the asymptotic performance of a per user unitary and rate control design. Other works like [4]-[5] studied transmit beamforming using different criteria and methods. However, in all these works, the focus was to study the quantization effects on the CSIT under noiseless feedback[1]. In practice, the CSIT feedback may not be error-free due to the feedback channel noise. Unlike the forward channel where heavy error correction coding can be applied to the time-interleaved payload, the limited CSIT feedback has to be received at the transmitter with minimum latency and hence, time interleaving is not possible. Furthermore, in most systems, the number of bits available for feedback is very limited (such as 6 bits in WiMAX) and hence, it will be more effective to utilize all the limited bits to carry the CSIT rather than wasting some bits to protect the CSIT feedback. The issue of noisy feedback is considered in [6], [7], [8], [9]. For example, in [6], [7], [8], the authors analyzed the effect of noisy feedback on the point-to-point MISO system and broadcast channel [9]. However, the authors did not incorporate the noisy feedback into the algorithm design. In [10], [11], the authors design a channel optimized quantizer for point-to-point MISO link to incorporate the noisy feedback link. However, extension

This paper is supported by RGC funding 615606.

[1]The CSIT feedback index is always received correctly at the transmitter.

to SDMA system is not-trivial. As illustrated in [12], the sensitivity of noisy feedback is much higher in SDMA systems and it is critical to take into consideration of noisy feedback in the robust limited feedback for SDMA systems.

When we have noisy CSIT feedback, there may be significant performance degradation because an erroneous CSIT feedback will make the base station selecting a wrong precoder for the user, which not only decreases the received signal to noise ratio (SNR) of the user but also increases the interference from other scheduled users since their assigned precoders are no longer orthogonal to the target user. As we shall illustrate in the paper, adopting a *naive design approach* (design the limited feedback codebook assuming error-free feedback and testing its performance in the noisy limited feedback situation) will result in very poor SDMA performance. In order to obtain a feedback-error-resilient design, there are several first-order technical challenges to be addressed:

- **Robust SDMA precoding & user selection:** Due to the noisy limited feedback, the CSIT index received at the base station may not be the same as that sent by the mobiles. As such, the selected precoder may not match the actual CSI at all, resulting in additional spatial interference among the selected SDMA users.
- **Robust rate adaptation:** To achieve a high system goodput advantage, robust rate adaptation is needed to control packet errors due to channel outage.
- **Robust Index Assignment:** Index assignment refers to the mapping of the CSIT feedback indices with the precoder entries in the codebook. With noisy feedback, CSIT index assignment plays an important role on the robustness performance of the SDMA systems.
- **Performance Analysis:** Beside robust limited feedback designs, it is important to have closed-form performance results to obtain useful design insights such as the sensitivity of CSIT errors in SDMA systems.

In this paper, we propose a robust noisy limited feedback design for SDMA systems. The solution consists of a real-time robust SDMA precoding, user selection and rate adaptation as well as an offline feedback index assignment algorithm. We formulate the robust index assignment problem into a Traveling Salesman Problem (TSP) and derive a low complex but asymptotically optimal solution. Simulation results show that the proposed framework has significant goodput gain compared to the traditional naive uncoded design (SDMA design assuming error-free and uncoded feedback) and naive coded design (SDMA design assuming error-free feedback but the limited CSIT feedback bits are protected by FEC) under noisy limited feedback channel. Furthermore, we show that despite the noisy feedback channels, the average system goodput of the proposed robust SDMA design scales as $\mathcal{O}(\frac{n_T(1-\epsilon)}{n_T-1}(C_{fb}-\log_2(N_n)))$ for interference limited scenario ($C_{fb} < (n_T-1)\log_2 P+\log_2 N_n$) and $\mathcal{O}(n_T\cdot\log_2 P)$ for noise limited scenario where n_T is the transmit antenna number, ϵ is the target outage probability, C_{fb} is the number of feedback bits, P is the transmission power and N_n is a constant. We find that in interference limited scenario, the average system goodput increase linearly with the number of feedback bits for fixed feedback SER and converge to a constant number for fixed feedback SNR. In the noise limited scenario, the average system goodput increase linearly with the number of transmit antenna and the forward channel SNR ($\log_2 P$).

II. System Model

In this paper, we shall adopt the following convention. $\mathbf{X}$ denotes a matrix and $\mathbf{x}$ denotes a vector. $\mathbf{X}^\dagger$ denotes matrix hermitian.

A. Forward MIMO Fading Channel Model

In this paper, we consider a multi-user system with a base station having n_T transmit antennas simultaneously transmitting to n_T one antenna active users from a total of K users. We shall focus on the case when $K > n_T$ so that user scheduling in addition to precoder adaptation is important. The base station separates n_T data streams to the active users by precoding. Each active user k is assigned a $n_T \times 1$ precoding vector $\mathbf{w}_k$. The precoder $\{\mathbf{w}_k\}_{k=1}^{n_T}$ are a set of unitary orthogonal vectors selected from a codebook of multiple sets of unitary

orthogonal vectors. Let x_k be the transmitted symbol of user k with $E[|x_k|^2] = 1$ and y_k denote the received symbol of user k. The forward channel is modeled as:

$$y_k = \sqrt{\frac{P}{n_T}} \mathbf{h}_k^\dagger \sum_{i \in \mathcal{A}} \mathbf{w}_i x_i + z_k \quad (1)$$

where P is the transmission power[2], $\mathbf{h}_k$ is the $n_T \times 1$ complex channel state vector of the k^{th} user, $\mathcal{A}$ is the active user set and z_k is the additive white Gaussian noise with zero-mean and unit variance. We assume that the transmit antennas and users are sufficient separated so that the channel fading between different users and different antenna are modeled as independent and identically distributed (i.i.d.) complex Gaussian process with zero-mean and unit variance[3].

We consider slow fading channels where the fading is quasi-static within a scheduling time slot for each user. This is a realistic assumption for pedestrian mobility (5km/hr) as the packet duration is of the order of 500ns (such as Wi-Fi and B3G). Due to the quasi static fading and noisy limited CSI feedback, there is uncertainty on a user's instantaneous mutual information (a function of the instantaneous CSI of all K users) at the transmitter. Hence, there exists potential packet errors (despite the use of powerful channel coding) due to channel outage when the transmitted data rate of user k exceeds its instantaneous mutual information.

B. Limited Feedback Processing at the Mobiles

In this paper, we consider FDD system and assume the CSI is estimated at each user (CSIR) perfectly and fed back to the base station through a feedback channel with a limited feedback capacity constraint C_{fb} bits per fading block per user. The CSIR of user k, $\mathbf{h}_k$, consists of two parts: channel gain $\|\mathbf{h}_k\| = \sqrt{\sum_{i=1}^{n_T} {h_k^i}^2}$ and channel shape $\tilde{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$. As will be explained in section III, the average system goodput is dominated by channel shape and channel gain has little influence on it especially in high SNR scenario[4]. Hence in this paper, we shall focus on utilizing all the C_{fb} feedback bits on the feedback of channel shape $\tilde{\mathbf{h}}_k$ and do not feed back channel gain $\|\mathbf{h}_k\|$.

We assume the K mobile stations quantize the channel shape of the local CSIR with a codebook consists of multiple sets of orthonormal vectors:

$$\mathcal{F} = \bigcup_{m=1}^{M} \mathcal{V}^{(m)} \quad (2)$$

where $\mathcal{F}$ denotes the quantization codebook with a cardinality $|\mathcal{F}| = N$, $\mathcal{V}^{(m)}$ is the m^{th} orthonormal set in the codebook and M is the number of orthonormal sets which is given by $M = \frac{N}{n_T}$. The M orthonormal bases of $\mathcal{F}$ are generated randomly and independently similar to [15][5]. Define the distortion function between two $n_T \times 1$ vectors $\mathbf{v}_1, \mathbf{v}_2$ as:

$$d(\mathbf{v}_1, \mathbf{v}_2) = 1 - |\mathbf{v}_1^\dagger \mathbf{v}_2|^2 = \sin^2(\angle(\mathbf{v}_1, \mathbf{v}_2)) \quad (3)$$

At the $k-th$ mobile, the quantized channel shape $\widehat{\widetilde{\mathbf{h}}}_k$ can be expressed as:

$$\widehat{\widetilde{\mathbf{h}}}_k = \arg\min_{\mathbf{v} \in \mathcal{F}} d(\mathbf{v}, \widetilde{\mathbf{h}}_k) \quad (4)$$

In order to reduce the total feedback overhead for all K users, user k shall decide whether to feed back its CSIT or not based on the criteria:

$$d(\widetilde{\mathbf{h}}_k, \widehat{\widetilde{\mathbf{h}}}_k) < \delta; \qquad \|\mathbf{h}_k\|^2 > g_{th} \quad (5)$$

where δ is the threshold for the distortion from actual channel shape to the quantized channel shape and g_{th} is the threshold for the channel gain. The motivation of (5) is allocating the feedback bits to the users with smaller CSI quantization error so as to reduce the potential spatial interference among the

[2]In this paper we assume equal power allocation since power allocation will only bring marginal performance gain under high SNR. This is also assumed in [2], [13], [14] etc.

[3]In this paper we assume the large scale fading parameters (path loss and shadowing) between a base station and all the users in the cell are the same.

[4]When there is only 1 active user in a cell, the system will degrade to a MISO system and this claim will be invalid. However, in MISO systems, interference is no longer a severe problem and there exists a lot of optimizing schemes. In this paper we will focus on strict SDMA system where there are more than 1 user served by the BS.

[5]Grassmannian codebook is not a good choice here. Maximizing the minimum distortion among M bases is not equivalent to maximizing the minimum distortion among all the vectors in the codebook.

SDMA streams. δ is a system parameter and can be selected offline. When the number of total user K is large, δ can be selected as a very small number to decrease the quantization error.

C. CSIT Feedback Channel Model

Unlike most of the previous literature where the limited feedback channel is assumed to be noiseless, we are interested in the more realistic case where the feedback channel may be noisy. Note that since the CSI feedback has to be delivered in a timely manner, effective FEC coding over many CSI feedbacks is not possible and hence, the feedback error cannot be ignored in practice.

The CSIT of user *k* is quantized locally and encoded into C_{fb} bits. The set of CSI indices sent at each of the K mobile stations $\{\mathcal{I}_k^{(MS)}\}$ and the corresponding CSI indices received at the base station $\{\mathcal{I}_k^{(BS)}\}$ both have cardinality of $N = 2^{C_{fb}}$. Assume the modulation symbol in the CSIT feedback channel has constellation $\mathcal{M}$ and the corresponding mapping from the CSIT index $\mathcal{I}_k^{(MS)}$ to the constellation point $\mathbf{m} \in \mathcal{M}$ is given by the 1-1 *index mapping function* $\mathcal{M} = \xi(\mathcal{I}_k^{(MS)})$. The probabilistic relationship between the CSIT feedback symbol sent $\xi(\mathcal{I}_k^{(MS)})$ and the CSIT feedback symbol received by the transmitter $\xi(\mathcal{I}_k^{(BS)})$ can be characterized by the *feedback channel transition matrix* $\mathbf{P}_{ch} = \{P^{ch}_{m_l,m_k}\}$, where:

$$P^{ch}_{m_l,m_k} = \Pr\left[\xi(\mathcal{I}_k^{(BS)}) = m_l | \xi(\mathcal{I}_k^{(MS)}) = m_k\right], m_l, m_k \in \mathcal{M}. \quad (6)$$

Hence, the *reliability* of the CSIT feedback channel is characterized by the CSIT feedback channel transition matrix $\mathbf{P}_{ch}$. Note that $\mathbf{P}_{ch}$ depends on the average feedback SNR, feedback constellation and so on and can be offline evaluated analytically or numerically through simulations.

Given $\mathbf{P}_{ch}$ and the index mapping rule ξ, the stochastic relationship between the CSIT index sent by the mobile station $\mathcal{I}_k^{(MS)}$ and the CSIT index received by the base station $\mathcal{I}_k^{(BS)}$ is characterized by the *CSIT index transition probability* $\mathbf{P}_{CSIT} = \{P^{CSIT}_{m_l,m_k}\}$ given by:

$$P^{CSIT}_{ij} = \Pr\left[\mathcal{I}_k^{(BS)} = j | \mathcal{I}_k^{(MS)} = i\right] = P^{ch}_{\xi(i),\xi(j)}, i, j \in [1, N]. \quad (7)$$

Suppose the condition in (5) is satisfied for the *k*-th mobile, the index of $\widehat{\widetilde{\mathbf{h}}}_k$, $\mathcal{I}_k^{(MS)}$, is then mapped to a constellation point m_k using an index mapping function $m_k = \xi(\mathcal{I}_k^{(MS)})$ and feed back to the base station. Due to the noisy feedback channel, selection of the index mapping function ξ becomes important and will affect the robustness of the SDMA system.

III. Base Station Processing: SDMA Precoding, User Scheduling and Rate Adaptation

In this section, we shall discuss the base station processing based on the limited feedback sent from the K mobiles over a noisy feedback channel with *index transition probability* $\mathbf{P}_{CSIT}$. Specifically, we shall discuss the SDMA precoding, user selection and rate adaptation.

A. System Goodput

Consider a full multiplexing system where base station schedules n_T active users for SDMA. Define $\mathcal{A}$ as the active user set and $\mathbf{w}_k$ as the precoder for a scheduled user *k*. We can write the instantaneous goodput (b/s/Hz successfully received by user *k*) ρ_k for user *k* as:

$$\rho_k = r_k(\mathcal{I}_k^{(BS)}) \cdot 1[r_k(\mathcal{I}_k^{(BS)}) < C_k(\mathbf{h}_k, \mathbf{w}_k)] \quad (8)$$

where r_k is the data rate of the packet of user *k* and is a function of received CSIT at base station, C_k is user *k*'s instantaneous mutual information and $1(A)$ is an indicator function which is 1 if the event A is true and 0 otherwise. User *k*'s instantaneous mutual information C_k is a function of its channel state $\mathbf{h}_k$ as well as the assigned precoder $\mathbf{w}_k$. Specifically, C_k can be written as:

$$C_k(\mathbf{h}_k, \mathbf{w}_k) = \log_2\left(1 + \frac{\frac{P}{n_T}|\mathbf{h}_k^\dagger \mathbf{w}_k|^2}{1 + \sum_{j \neq k, j \in \mathcal{A}} \frac{P}{n_T}|\mathbf{h}_k^\dagger \mathbf{w}_j|^2}\right) \quad (9)$$

We define $\theta = \angle(\widetilde{\mathbf{h}}_k, \mathbf{w}_k)$ as the angle between the actual channel shape and the assigned precoder for user *k*. Similarly, define $\phi = \angle(\widetilde{\mathbf{h}}_k, \mathbf{v}_{\mathcal{I}_k^{(MS)}})$ and $\varphi(\mathbf{v}_i, \mathbf{v}_j) = \angle(\mathbf{v}_i, \mathbf{v}_j)$ as the angle between the actual channel shape and the quantized vector of user *k* and the angle between two $n_T \times 1$ vectors $\mathbf{v}_i, \mathbf{v}_j$.

Consider high effective SNR asymptotic scenario where $\frac{P}{n_T}\|\mathbf{h}_k\|^2$ is sufficiently large. We can further simplify (9) to:

$$C_k \approx \log_2(1+\frac{\cos^2\theta}{\sum_{j\neq k,j\in\mathcal{A}}\cos^2\varphi(\widetilde{\mathbf{h}}_k,\mathbf{w}_j)}) \quad (10)$$

As shown in Figure 1, the approximation is quite good for moderate to high SNR.

Remark 1: While the approximation in (10) fails when $\sum_{j\neq k,j\in\mathcal{A}}\cos^2\varphi(\widetilde{\mathbf{h}}_k,\mathbf{w}_j)\rightarrow 0$, it will not affect our design and analysis because for a practical target PER (e.g. 10^{-2}), when $\sum_{j\neq k,j\in\mathcal{A}}\cos^2\varphi(\widetilde{\mathbf{h}}_k,\mathbf{w}_j)\rightarrow 0$, both C_k in (9) and (10) will be large enough and no outage will occur. In other words, the case $\sum_{j\neq k,j\in\mathcal{A}}\cos^2\varphi(\widetilde{\mathbf{h}}_k,\mathbf{w}_j)\rightarrow 0$ will not be the performance bottleneck and there is no loss of generality to focus on the bottleneck case when $\sum_{j\neq k,j\in\mathcal{A}}\cos^2\varphi(\widetilde{\mathbf{h}}_k,\mathbf{w}_j)$ is not close to 0.

Remark 2: From equation (10), we can see that at high SNR, the instantaneous mutual information does not depend on the channel gain $\|\mathbf{h}_k\|$ and hence the average system goodput at high SNR is dominated by channel shape.

The average goodput for a scheduled user *k* can be expressed as:

$$\begin{aligned}\overline{\rho}_k &= \mathbb{E}_{\mathcal{I}_k^{(BS)}}\mathbb{E}_{\mathbf{h}_k}[\rho_k]\\ &= \mathbb{E}_{\mathcal{I}_k^{(BS)}}[r_k(\mathcal{I}_k^{(BS)})\cdot Pr(r_k<C_k|\mathcal{I}_k^{(BS)})] \quad (11)\end{aligned}$$

In practice, there is a target PER requirement ϵ associated with different application streams (e.g. $\epsilon=0.01$ for voice applications), which is expressed as:

$$1-Pr(r_k(\mathcal{I}_k^{(BS)})<C_k|\mathcal{I}_k^{(BS)})=\epsilon \quad (12)$$

The average system goodput for all scheduled users can be written as:

$$\mathcal{G} = \sum_{k\in\mathcal{A}}\overline{\rho}_k = n_T\cdot(1-\epsilon)\cdot\mathbb{E}_{\mathcal{I}_k^{(BS)}}[r_k(\mathcal{I}_k^{(BS)})] \quad (13)$$

B. SDMA Precoding and User Selection

Generally speaking, based on the CSIT feedback, the active user set $\mathcal{A}$ and corresponding precoder $\mathbf{w}_k$ shall be jointly optimized to maximize the overall average system goodput in (13). Assume that the number of total user K is large enough such that the base station can always fully schedule n_T active users. A natural algorithm includes an exhaustive search over all the combinations of n_T users out of a pool of K users and jointly optimize the n_T precoders for the selected users to maximize the average system goodput. However, exhaustive search has exponential complexity in terms of number of users and is not practical as an online algorithm especially for large number of users. Therefore we shall adopt a simple orthogonal scheduling and precoding algorithm. At base station, take the received CSIT $\mathbf{v}_{\mathcal{I}_k^{(BS)}}$ as the estimation of $\widetilde{\mathbf{h}}_k$, the term $\cos^2\theta$ in the signal power term in (10) is maximized if $\mathbf{w}_k=\mathbf{v}_{\mathcal{I}_k^{(BS)}}$. Similarly, the interference term $\sum_{j\neq k,j\in\mathcal{A}}\cos^2\varphi(\widetilde{\mathbf{h}}_k,\mathbf{w}_j)$ is minimized by choosing $\mathbf{w}_j\perp\mathbf{v}_{\mathcal{I}_k^{(BS)}},\ \ \forall\ j\neq k, j\in\mathcal{A}$.

It can be easily shown that the above two equations can be satisfied simultaneously with:

$$\mathbf{v}_{\mathcal{I}_k^{(BS)}}\perp\mathbf{v}_{\mathcal{I}_j^{(BS)}},\ \ \forall j\neq k, j,k\in\mathcal{A}. \quad (14)$$

In other words, the received CSIT of n_T scheduled users $\{\mathbf{v}_{\mathcal{I}_k^{(BS)}}\}$ form an orthonormal set $\mathcal{V}^{(m)}$ in the codebook $\mathcal{F}$. In fact, SDMA with orthogonal precoding is also called *per unitary basis stream user and rate control* (PU2RC) [16] and has been widely used in the standards such as 3GPP-LTE[3]. The main feature of PU2RC is that it could accommodate limited CSIT feedback in a natural way. For instance, the multiuser precoders are selected from a codebook of multiple orthonormal bases. The importance of PU2RC for the next-generation wireless communication motivates the investigation of its performance in this paper. In this paper, we consider a simplified PU2RC system where scheduled users have single data streams, which are separated by orthogonal precoders.

The precoding and user scheduling strategy is briefly summarized as follows:

- *Step 1: search N/n_T groups of orthonormal sets $\mathcal{V}^{(m)}$ in $\mathcal{F}$ for one set $\mathcal{V}^{(m)*}$ in which each vector is received by the base station as a CSIT feedback for at least one user. If there exists multiple satisfying sets, randomly pick one.*
- *Step 2: For an orthonormal vector $\mathbf{v}_k\in\mathcal{V}^{(m)*}$, randomly select one user from the group of users with received CSIT $\mathbf{v}_k$ as the scheduled user and set $\mathbf{v}_k$ as its precoder.*

- *Step 3: Repeat Step 2 for all the vectors in* $\mathcal{V}^{(m)*}$

C. Robust Rate Adaptation

With the proposed precoding and user scheduling strategy, C_k in (10) is given by the following lemma:

Lemma 1 (Mutual Information at High SNR)**.** *At high downlink SNR, when the n_T scheduled users are using n_T orthonormal precoders to transmit, the mutual information of a scheduled user is given by:*

$$C_k = -2\log_2(\sin\theta). \tag{15}$$

Proof 1. *Please refer to Appendix A for details.*

r_k can be calculated from the requirement of the conditional PER target in (12), which is given by:

$$P_{out} = Pr(-2\log_2(\sin\theta) < r_k|\mathcal{I}_k^{(BS)}) = \epsilon \tag{16}$$

Yet, one critical challenge in solving for r_k in (16) is the knowledge of the CDF of $\sin\theta$ conditioned on the CSIT $\mathcal{I}_k^{(BS)}$. This is in contrast with the conventional approach of maximizing the ergodic capacity in which only the first order moment of the random variable $\sin\theta$ is needed. In the following lemma, we shall give a tight upper bound on the conditional CDF of $\sin\theta$, which is critical to solving for a closed-form rate adaptation solution.

Lemma 2 (Upper bound of the PER P_{out})**.** *The conditional PER P_{out} of the forward channel (conditioned on the limited CSIT feedback received at the BS) is given by:*

$$\begin{aligned} P_{out} &\leq (1 - \frac{(2^{-\frac{r_k}{2}} - \sin\varphi_{\mathcal{I}_k^{(BS)},i^*})^{2(n_T-1)}}{\delta^{(n_T-1)}}) \cdot P^{CSIT}_{i^*,\mathcal{I}_k^{(BS)}} \\ &+ \sum_{j\in\overline{Ns^{\epsilon}}_{\mathcal{I}_k^{(BS)}}} P^{CSIT}_{j,\mathcal{I}_k^{(BS)}} \end{aligned} \tag{17}$$

where $i^* = \arg\max_{i\in Ns^{\epsilon}(\mathcal{I}_k^{(BS)})}\sin\varphi_{\mathcal{I}_k^{(BS)},i}$ and $Ns^{\epsilon}(\mathcal{I}_k^{(BS)})$ is the set of neighboring codewords of $\mathcal{I}_k^{(BS)}$ satisfying: $\sum_{j\in Ns^{\epsilon}(\mathcal{I}_k^{(BS)})} P^{CSIT}_{j,\mathcal{I}_k^{(BS)}} \geq 1-\epsilon$ and $\sum_{j\in Ns^{\epsilon}(\mathcal{I}_k^{(BS)})} P^{CSIT}_{j,\mathcal{I}_k^{(BS)}} - P^{CSIT}_{i^*,\mathcal{I}_k^{(BS)}} < 1-\epsilon$.

Proof 2. *Please refer to Appendix B for details.*

Note that in practice, i^* and $Ns^{\epsilon}(\mathcal{I}_k^{(BS)})$ can be offline pre-calculated from the channel transition matrix P^{CSIT} and the distortion between the codewords $\sin\varphi_{i,j}$. For example, suppose our codebook is given by $\mathbf{F} = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$. Assume $\epsilon = 0.1$ and $P^{CSIT}_{11} = 0.70, \sin\angle(\mathbf{v_1}, \mathbf{v_1}) = 0; P^{CSIT}_{21} = 0.10, \sin\angle(\mathbf{v_1}, \mathbf{v_2}) = 0.5; P^{CSIT}_{31} = 0.11, \sin\angle(\mathbf{v_1}, \mathbf{v_3}) = 0.4; P^{CSIT}_{41} = 0.09, \sin\angle(\mathbf{v_1}, \mathbf{v_4}) = 1$, then $Ns^{0.1}(1) = \{\mathbf{v_2}, \mathbf{v_3}\}$ and $i^* = 3$.

Using Lemma 2, and define $\epsilon_{res} = \epsilon - \sum_{j\in\overline{Ns^{\epsilon}}_{\mathcal{I}_k^{(BS)}}} P^{CSIT}_{j,\mathcal{I}_k^{(BS)}}$, the transmission rate of user *k* is given by:

$$r_k = -2\log_2\left\{\delta^{\frac{1}{2}}(1 - \frac{\epsilon_{res}}{P^{CSIT}_{i^*,\mathcal{I}_k^{(BS)}}})^{\frac{1}{2(n_T-1)}} + \sin\varphi_{\mathcal{I}_k^{(BS)},i^*}\right\}. \tag{18}$$

IV. Robust Index Assignment

Index mapping algorithm is important when there is noise on feedback channel. In this section, we shall optimize the index assignment function ξ to maximize the system goodput in (13). This is equivalent to minimize the feedback distortion between the channel shape $\widetilde{\mathbf{h}}_k$ and corresponding precoder of user k $\mathbf{v}_k^{(BS)}$, which is $\sin^2\theta$. This feedback distortion is contributed by two parts: distortion from quantization $\sin^2(\phi)$ and distortion from feedback error $\sin^2(\varphi(\mathbf{v}_k^{(BS)}, \mathbf{v}_k^{(MS)}))$. Based on:

$$\varphi_{\mathcal{I}_k^{(BS)},\mathcal{I}_k^{(MS)}} - \phi \leq \theta \leq \varphi_{\mathcal{I}_k^{(BS)},\mathcal{I}_k^{(MS)}} + \phi \tag{19}$$

and $\sin^2\phi < \delta$ where δ is chosen to be a small value to avoid excessive spatial interference among the SDMA streams. As a result,we shall omit quantization distortion $\sin^2\phi$ and focus on minimizing the distortion introduced by feedback error $d(\mathbf{v}_k^{(BS)}, \mathbf{v}_k^{(MS)}) = \sin^2(\varphi(\mathbf{v}_k^{(BS)}, \mathbf{v}_k^{(MS)}))$. The average distortion is given by:

$$\mathbb{E}(d) = \sum_{i=1}^{N}\sum_{j=1}^{N} Pr(\mathbf{v}_i)\cdot P^{ch}_{\xi(i),\xi(j)}\cdot d(\mathbf{v}_i, \mathbf{v}_j) \tag{20}$$

Searching for optimal index mapping function ξ can be summarized into the following problem:

Problem 1 (Robust Index Assignment Problem)**.** *Find an optimal index assignment function to minimize the average distortion introduced by feedback error:*

$$\xi^* = \arg\min_{\xi} \sum_{i=1}^{N}\sum_{j=1}^{N} Pr(\mathbf{v}_i)\cdot P^{ch}_{\xi(i),\xi(j)}\cdot d(\mathbf{v}_i,\mathbf{v}_j) \qquad (21)$$

where $Pr(\mathbf{v}_i)$ is the probability that $\mathbf{v}_i$ is the quantization output for $\mathbf{h}_k$ and $d(\mathbf{v}_i,\mathbf{v}_j)$ is the distortion between two codeword given in (3).

Note that the insight of the above formulation is that a good index mapping function shall map 2 precoders $\mathbf{v}_1,\mathbf{v}_2 \in \mathcal{F}$ with smaller distortion $d(\mathbf{v}_1,\mathbf{v}_2)$ to the constellation points m_1, m_2 with larger transition probability $P^{ch}_{m_1,m_2}$.

In general, finding the optimal mapping $\xi(.)$ involves combinatorial search. When the number of feedback bits is small, the computation complexity of exhaustive search is still acceptable. However, when the number of transmit and receive antennas gets larger and more feedback bits are required, the exhaustive searching time will increase double exponentially with C_{fb}. This motivates the study on the low-complexity solution of the problem.

Consider a special case when the CSIT feedback index is modulated by one N-PSK symbol. When feedback error occurs, the erroneous symbol is likely to be one of the adjacent neighbors of the feedback N-PSK symbol, which is referred as the nearest constellation error. The average distortion introduced by feedback error in (21) can be simplified to:

$$\begin{aligned} D(\xi) &= P_e\cdot\{(\frac{Pr(\mathbf{v}_{\xi^{-1}(1)})d(\mathbf{v}_{\xi^{-1}(1)},\mathbf{v}_{\xi^{-1}(2)})}{2} + \\ &\quad \frac{Pr(\mathbf{v}_{\xi^{-1}(2)})d(\mathbf{v}_{\xi^{-1}(2)},\mathbf{v}_{\xi^{-1}(1)})}{2}) + \cdots + \\ &\quad (\frac{Pr(\mathbf{v}_{\xi^{-1}(N)})d(\mathbf{v}_{\xi^{-1}(N)},\mathbf{v}_{\xi^{-1}(1)})}{2} + \\ &\quad \frac{Pr(\mathbf{v}_{\xi^{-1}(1)})d(\mathbf{v}_{\xi^{-1}(1)},\mathbf{v}_{\xi^{-1}(1)})}{2})\} \end{aligned} \qquad (22)$$

where P_e denotes the symbol error rate (SER) of the feedback channel and $D(\xi) = \mathbb{E}(d)$ is the average distortion. Assume that $\{Nd_1,\cdots,Nd_N\}$ are N virtual cities, and the distance between the virtual cities Nd_i and Nd_j is given by:

$$Dis(Nd_i,Nd_j) = P_e\frac{Pr(\mathbf{v}_i)d(\mathbf{v}_i,\mathbf{v}_j)+Pr(\mathbf{v}_j)d(\mathbf{v}_j,\mathbf{v}_i)}{2} \qquad (23)$$

Equation (22) can be expressed in terms of distance between *virtual cities* as follows.

$$\begin{aligned} D(\xi) &= Dis(Nd_{\xi^{-1}(N)},Nd_{\xi^{-1}(1)}) + \\ &\quad Dis(Nd_{\xi^{-1}(1)},Nd_{\xi^{-1}(2)}) + \cdots + \\ &\quad Dis(Nd_{\xi^{-1}(N-1)},Nd_{\xi^{-1}(N)}) \end{aligned} \qquad (24)$$

Hence, the optimization metric is equivalent to the total distance of a Hamiltonian cycle [17]. From equation (24), the index mapping problem in Problem 1 is equivalent to searching shortest path in a Hamiltonian cycle and this can be cast into a *traveling salesman problem* (TSP). This is summarized below.

Problem 2 (Traveling Salesman Problem)**.** *Given a number of cities $\{Nd_1,Nd_2,\cdots,Nd_N\}$, and the costs of traveling from any city to any other city $\{Dis(Nd_i,Nd_j)\}$, what is the round-trip route $[\xi^{-1}(1),\xi^{-1}(2),\cdots,\xi^{-1}(N)]$ that visits each city exactly once and then returns to the starting city to minimize the total distance (24).*

TSP is found to be an NP-hard (nondeterministic polynomial time) problem and yet, there are a number of efficient searching algorithms for the TSP such as the cutting-plane method [18] and genetic algorithm[19].

In this paper, we propose a simple construction algorithm, namely the *Circled Nearest Neighbor Algorithm*(CNNA). The CNNA algorithm is described below:

- *Step 1: Start the TSP travel from a randomly selected node Nd_i.*
- *Step 2: Go to the nearest unvisited node from Nd_i. If there exists more than one such nodes, we select the one with smallest sum distortion to the previously visited nodes. For example, if $\mathcal{S}$ is the set of nodes already visited, and $\mathcal{N}_i$ is the set of unvisited neighboring nodes of Nd_i, we shall select Nd_j via the criteria: $Nd_j = \arg\min_{Nd_j\in\mathcal{N}_i}\sum_{Nd_k\in\mathcal{S}} dis(Nd_k,Nd_j)$*
- *Step 3: Repeat Step 2 till all the nodes are visited. Then go back to the start node.*

Suppose we require that all feedback error results in the "nearest neighboring precoder" and we would like to find an index assignment such that this can be realized for "2-nearest

neighbor error feedback channel". To do that, we assume all the neighboring regions of codewords are "equi-probable" and isotropically distributed on the surface of a unitary hypersphere ($\widetilde{\mathbf{h}}_k$-space). For the given topology of the partition region, the above algorithm of index assignment will result in a circled pattern as illustrated in figure 2 and is similar to the following analogy. Suppose we start from north pole of the Earth and travel around the world along the latitudes to the south pole. When we finish traveling along a latitude, we go down to the next one until we reach the south pole. After south pole is arrived, we go back to the north pole directly. Define d_{min} as the minimum distortion between two precoder, we have the following lemma:

Lemma 3 (Asymptotic Optimality of CNNA Algorithm)**.** *For N-PSK constellation with nearest-constellation error approximation, the index mapping solution given by the CNNA algorithm ξ^* is asymptotically optimal for sufficiently large N. i.e.*

$$\lim_{N\to+\infty}\frac{D(\xi^*)}{N} = d_{min}. \tag{25}$$

Proof 3. *We provide a sketch of proof due to page limit. With the proposed algorithm, all error events shall result in nearest neighbor codeword errors (for 2-nearest constellation feedback channel) except for the codewords serving as the starting and ending nodes. Hence we have*

$$D = N\cdot d_{min} + c \tag{26}$$

where c is a constant. When N is sufficiently large, the average distortion of the travel is $\lim_{N\to+\infty}\frac{D}{N} = d_{min}$. Hence the proposed algorithm is asymptotically optimal.

V. Performance Analysis

In this section, we shall focus on obtaining the asymptotic goodput performance under noisy limited feedback. The transmission rate for a scheduled user in (18) can be bounded by: $-2\log_2(\delta^{\frac{1}{2}} + \sin\varphi_{\mathcal{I}_k^{(BS)},i^*}) \le r_k \le -2\log_2(\sin\varphi_{\mathcal{I}_k^{(BS)},i^*})$.

In fact, the upper bound and lower bound of r_k above are both very tight when δ is small. Asymptotically, the two bounds will meet each other as the number of feedback bits C_{fb} goes to infinity since δ will approach to 0 and $\sin\varphi_{\mathcal{I}_k^{(BS)},i^*}$ is the dominant factor to r_k. Since we are interested in the first-order analysis, the data rate r_k can be taken as:

$$r_k = \mathcal{O}(-2\log_2(\sin\varphi_{\mathcal{I}_k^{(BS)},i^*})) \tag{27}$$

Substitute (27) into (13), the average system goodput is given by:

$$\mathcal{G} \;=\; \mathcal{O}(-2n_T(1-\epsilon)\log_2(\mathbb{E}_{\mathcal{I}_k^{(BS)}}[\sin\varphi_{\mathcal{I}_k^{(BS)},i^*}])) \tag{28}$$

From (28), we can see that system goodput performance depends on the worst case distortion $\sin\varphi_{\mathcal{I}_k^{(BS)},i^*}$ of an index assignment function ξ. Define $N_n^\epsilon(i)$ as the set of neighboring points of a constellation point i (including itself) with $\sum_{j\in N_n^\epsilon(i)} P_{ch}(i,j) = 1-\epsilon$. The cardinality of the set is $N_n = |N_n^\epsilon(i)|$. Note that $N_n^\epsilon(i)$ is the N_n largest terms in the i_th row of P_{ch}.[6] As a result, both P_e and N_n are two first order parameters to characterize the quality of feedback channel. We have the following lemma:

Lemma 4 (Lower bound for $\mathbb{E}_{\mathcal{I}_k^{(BS)}}(\sin\varphi_{\mathcal{I}_k^{(BS)},i^*})$)**.** *For sufficiently large N, we have:*

$$\mathbb{E}_{\mathcal{I}_k^{(BS)}}(\sin\varphi_{\mathcal{I}_k^{(BS)},i^*}) \ge (\frac{N_n}{N})^{\frac{1}{2(n_T-1)}} \tag{29}$$

Proof 4. *Please refer to Appendix C for details.*

Numerical results show that with optimal or near optimal index assignment, the lower bound is quite tight as illustrated in figure 3. Take the lower bound in (29) as an approximation of $\mathbb{E}_{\mathcal{I}_k^{(BS)}}(\sin\varphi_{\mathcal{I}_k^{(BS)},i^*})$ and substitute into (28), we have the following theorem:

Theorem 1 (Asymptotic System Goodput $\mathcal{G}$ in Interference Dominant Scenario)**.** *When the number of feedback bits C_{fb} satisfies $C_{fb} < (n_T-1)\log_2 P + \log_2 N_n$, the system is*

[6]Example: For 8PSK with 10dB feedback SNR we have $N_n = 3$ when $\epsilon = 0.03$.

dominated by interference. For sufficiently large transmission power P and quantization codebook size N, the average system goodput is given by:

$$\mathcal{G} = \mathcal{O}(\frac{n_T(1-\epsilon)}{n_T-1}(C_{fb}-\log_2(N_n))) \quad (30)$$

Corollory 1. *In interference dominant system with sufficiently large transmission power P and quantization codebook size N:*

- *With fixed SER P_e on the feedback channel, N_n is a finite constant and the average system goodput is given by:*

$$\mathcal{G} = \mathcal{O}(\frac{n_T(1-\epsilon)}{n_T-1}C_{fb}) \quad (31)$$

- *With fixed feedback SNR, N_n scales with N as $N_n = c \cdot 2^{C_{fb}}$ where c is a constant and the average system goodput is given by:*

$$\mathcal{G} = \mathcal{O}(\frac{n_T(1-\epsilon)}{n_T-1}) \quad (32)$$

The goodput order of growth results in (31) and (32) are also verified against simulations in Figure 4 and Figure 5.

On the other hand, when the number of feedback bits is sufficiently large, the SDMA system will operate in the noise-dominated regime.

Theorem 2 (Asymptotic System Goodput $\mathcal{G}$ in Noise Dominant Scenario)**.** *When the number of feedback bits C_{fb} satisfies $C_{fb} > (n_T-1)\log_2 P + \log_2 N_n$, the system is noise-dominated. For sufficiently large transmission power P and quantization codebook size N, the system goodput is given by:*

$$\mathcal{G} = \mathcal{O}(n_T \cdot \log_2 P) \quad (33)$$

Proof 5. *Please refer to Appendix D for details.*

The above result also reduces to that for SDMA system with noiseless feedback when $N_n = \mathcal{O}(1)$. Note that $N_n = \mathcal{O}(1)$ actually corresponds to an asymptotically noiseless feedback channel. We shall note that $C_{fb} > (n_T-1)\log_2 P + \log_2 N_n$ may not be able to be satisfied by simply increasing C_{fb} since C_{fb} will be canceled when $N_n = c \cdot 2^{C_{fb}}$ (e.g. constant feedback SNR). This indicates that one could not enhance the CSIT quality by increasing C_{fb} if the feedback SNR is kept constant.

VI. Results and Discussions

In this section, we study the performance of the proposed robust SDMA system under noisy limited feedback. We compare the performance of proposed design with two naive designs under the same feedback cost. In the uncoded naive design, the SDMA system is designed as if the limited CSIT feedback were noiseless and the limited CSIT feedback bits are uncoded. In the coded naive design, the SDMA system is similar to the uncoded naive design except that the limited CSIT feedback bits are protected by hamming code. In the simulations, we consider an SDMA system with $n_T = 4$ forward SNR 20dB and $K = 100$. We set the thresholds $\delta = 0.1$ and $g_{th} = 2$.

A. System Performance with respect to the Number of Feedback Bits C_{fb}

Figure 4 illustrates the average goodput versus the number of feedback bits of the SDMA system with a fixed feedback SER of 0.2[7]. There are significant goodput gain compared with both the naive uncoded and coded designs. The order-of-growth expressions in Theorem 1 and Corollary 1 are also verified.

Figure 5 illustrates the average goodput versus the modulation level per feedback symbol with a fixed average feedback SNR of 10dB and a fixed number of feedback symbols. There are also significant goodput gain in the proposed scheme compared with both the naive uncoded and coded designs. In addition, there is a tradeoff relationship in the feedback constellation level for the baseline systems. With lower feedback constellation level (such as BPSK), the feedback is more robust but the average goodput performance is limited by the resolution in the CSI feedback. On the other hand, for large feedback constellation, the average goodput performance of the reference baselines are poor because the performance is limited by the feedback error.

[7]This corresponds to a feedback SNR of 10 dB when $C_{fb} = 3$ and 8PSK is adopted.

B. System Performance with respect to Feedback Quality

The feedback quality can be specified by feedback SER and feedback SNR. Figure 6 shows the average system goodput versus SNR with different feedback SER and fixed feedback bits $C_{fb} = 8$. It is shown that with the proposed design, the system goodput decrease much slower with the increasing of the feedback SER. In figure 7, we show the average system goodput versus feedback SNR with fixed feedback bits $C_{fb} = 6$. With proposed design the system goodput increases much faster with the increasing of the feedback SNR.

VII. CONCLUSION

In this paper, we proposed a robust noisy limited feedback design with a joint user scheduling and precoder scheme as well as rate adaptation and robust index assignment optimization algorithms. We convert the index assignment optimization problem to a *Traveling Salesman Problem*(TSP). Simulation results show that the proposed framework has significant goodput gain compared to the uncoded and coded naive designs. Furthermore, we show that despite the noisy feedback, the average system goodput scales as $\mathcal{O}(\frac{n_T(1-\epsilon)}{n_T-1}(C_{fb} - \log_2(N_n)))$ and $\mathcal{O}(n_T \cdot \log_2 P)$ in the interference limited regime ($C_{fb} < (n_T-1)\log_2 P + \log_2 N_n$) and noise-limited regime respectively.

APPENDIX-A: PROOF OF LEMMA 1

From equation (9), the instantaneous mutual information can be written as:

$$\begin{aligned} C_k(\mathbf{h}_k, \mathbf{w}_k) &\approx \log_2(1 + \frac{\cos^2\theta}{\sum_{j\neq k, j\in\mathcal{A}} \cos^2\varphi(\widetilde{\mathbf{h}}_k, \mathbf{w}_j)}) \\ &= -2\log_2(\sin\theta) \end{aligned} \tag{34}$$

(34) is because all the scheduled precoders $\mathbf{w}_j$ forms an orthogonal bases of the n_T dimensional space and $(\cos\varphi(\widetilde{\mathbf{h}}_k, \mathbf{w}_{j1}), \cdots, \cos\varphi(\widetilde{\mathbf{h}}_k, \mathbf{w}_{jn_T}))$ is a vector on the unit sphere of the n_T dimensional space. where $\mathbf{w}_{j1}$ to $\mathbf{w}_{jn_T}$ are n_T scheduled orthonomal precoders. Hence we have:

$$\cos^2\theta + \sum_{j\neq k, j\in\mathcal{A}} \cos^2\varphi(\widetilde{\mathbf{h}}_k, \mathbf{w}_j)) = 1. \tag{35}$$

APPENDIX-B: PROOF OF LEMMA 2

Given received CSIT $\mathcal{I}_k^{(BS)}$, the outage probability can be written as:

$$\begin{aligned} P_{out} &\approx \frac{Pr(2^{-\frac{r_k}{2}} - \sin\varphi_{\mathcal{I}_k^{(BS)}, i^*} < \sin\phi \le \sqrt{\delta})}{Pr(\sin\phi < \sqrt{\delta})} \cdot P^{CSIT}_{i^*, \mathcal{I}_k^{(BS)}} \\ &\quad + \sum_{j\in \overline{Ns^{\epsilon}}_{\mathcal{I}_k^{(BS)}}} P^{CSIT}_{j, \mathcal{I}_k^{(BS)}} \end{aligned} \tag{36}$$

where $i^* = \arg\max_{i\in Ns^{\epsilon}(\mathcal{I}_k^{(BS)})} \sin\varphi_{\mathcal{I}_k^{(BS)}, i}$ and $Ns^{\epsilon}(\mathcal{I}_k^{(BS)})$ is the set of neighboring codewords of $\mathcal{I}_k^{(BS)}$ satisfying: $\sum_{j\in Ns^{\epsilon}(\mathcal{I}_k^{(BS)})} P^{CSIT}_{j, \mathcal{I}_k^{(BS)}} \ge 1-\epsilon$ and $\sum_{j\in Ns^{\epsilon}(\mathcal{I}_k^{(BS)})} P^{CSIT}_{j, \mathcal{I}_k^{(BS)}} - P^{CSIT}_{i^*, \mathcal{I}_k^{(BS)}} < 1-\epsilon$. From [20], we have the CDF for $\sin\phi$:

$$Pr(\sin\phi < x) = x^{2(n_T-1)} \tag{37}$$

Subscribing (37) into (36), we can simplify the outage probability as:

$$\begin{aligned} P_{out} &\le (1 - \frac{(2^{-\frac{r_k}{2}} - \sin\varphi_{\mathcal{I}_k^{(BS)}, i^*})^{2(n_T-1)}}{\delta^{(n_T-1)}}) \cdot P^{CSIT}_{i^*, \mathcal{I}_k^{(BS)}} \\ &\quad + \sum_{j\in \overline{Ns^{\epsilon}}_{\mathcal{I}_k^{(BS)}}} P^{CSIT}_{j, \mathcal{I}_k^{(BS)}}. \end{aligned}$$

APPENDIX-C: PROOF OF LEMMA 4

The asymptotic expression for $\mathbb{E}_{\mathcal{I}_k^{(BS)}}(\sin\varphi_{\mathcal{I}_k^{(BS)}, i^*})$ is deduced assuming all the channel shape quantized to the elements in $Ns^{\epsilon}(\mathcal{I}_k^{(BS)})$ forms a neighboring region of $\mathbf{v}_{\mathcal{I}_k^{(BS)}}$ defined as a hypersphere:

$$\mathcal{C}_b(\mathbf{v}_{\mathcal{I}_k^{(BS)}}) = \{\widetilde{\mathbf{h}}_k : \sin\angle(\widetilde{\mathbf{h}}_k, \mathbf{v}_{\mathcal{I}_k^{(BS)}}) \le r_{\mathcal{C}_b}\} \tag{38}$$

where $r_{\mathcal{C}_b}$ is the radius of the hypersphere and $\mathbf{v}_{i^*}$ lies at the edge of the hypersphere. Since $Pr(\sin\angle(\widetilde{\mathbf{h}}_k, \mathbf{v}_i) < x) = x^{2(n_T-1)}$, and the size of the hypersphere is:

$$Pr(\sin\angle(\widetilde{\mathbf{h}}_k, \mathbf{v}_{\mathcal{I}_k^{(BS)}}) \le r_{\mathcal{C}_b}) = \frac{|Ns^{\epsilon}(\mathcal{I}_k^{(BS)})|}{N} \ge \frac{N_n}{N} \tag{39}$$

where $|Ns^{\epsilon}(\mathcal{I}_k^{(BS)})|$ is the cardinality of $Ns^{\epsilon}(\mathcal{I}_k^{(BS)})$. Therefore we have $r_{\mathcal{C}_b} \ge (\frac{N_n}{N})^{\frac{1}{2(n_T-1)}}$

Since $\mathbf{v}_{i^*}$ lies at the edge of the hypersphere, we have:

$$\mathbb{E}_{\mathcal{I}_k^{(BS)}}(\sin\varphi_{\mathcal{I}_k^{(BS)}, i^*}) \ge (\frac{N_n}{N})^{\frac{1}{2(n_T-1)}}.$$

APPENDIX-D: PROOF OF THEOREM 2

Consider the situation with noise power larger than interference power $1 > \mathbb{E}[\frac{P}{n_T}\|\mathbf{h}_k\|^2 \sin^2\theta]$ as noise dominant scenario. With optimal or near optimal index assignment, we have $\mathbb{E}[\sin^2\theta] \leq (\frac{N_n}{N})^{\frac{1}{n_T-1}}$. Therefore, we require $P \cdot (\frac{N_n}{N})^{\frac{1}{n_T-1}} < 1$ which gives $C_{fb} > (n_T-1)\log_2 P + \log_2 N_n$. The instantaneous mutual information of a user in (9) can be simplified into $C_k \approx \log_2(1+\frac{P}{n_T}\|h_k\|^2\cos^2\theta)$. As the channel gain $\|\mathbf{h}_k\|^2$ scale with n_T, without loss of generality, we can also select g_{th} in (5) at order $O(n_T)$. Therefore the transmission rate for user k has the order $r_k \sim O(\log_2 n_T)$ hence the system goodput $\mathcal{G}$ has the order:

$$\mathcal{G} \quad \sim \quad O(n_T \cdot \log_2 P). \tag{40}$$

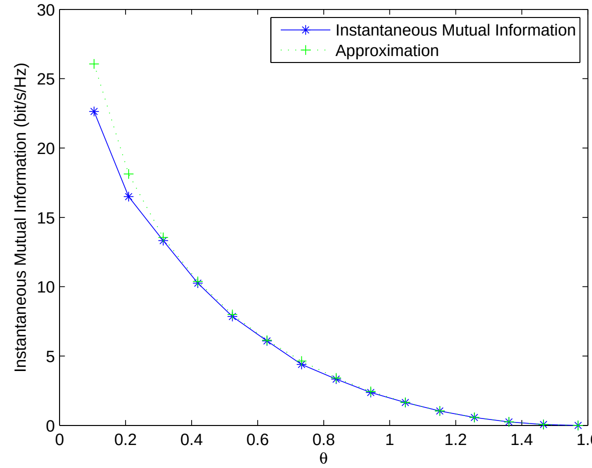

Fig. 1. Accuracy of approximation on instantaneous mutual information at $SNR = 20dB$ and different θ in equation 10.

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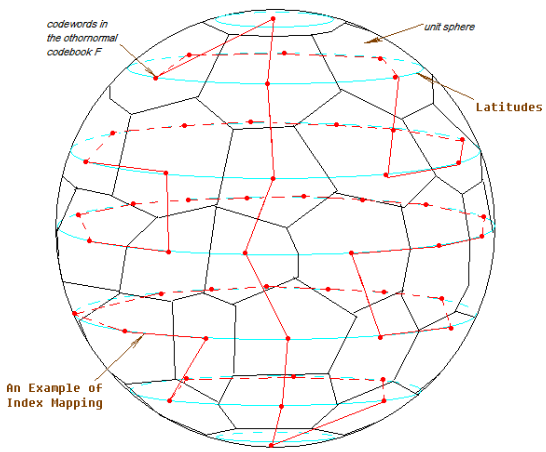


Fig. 2. A heuristic TSP travel on a hypersphere.

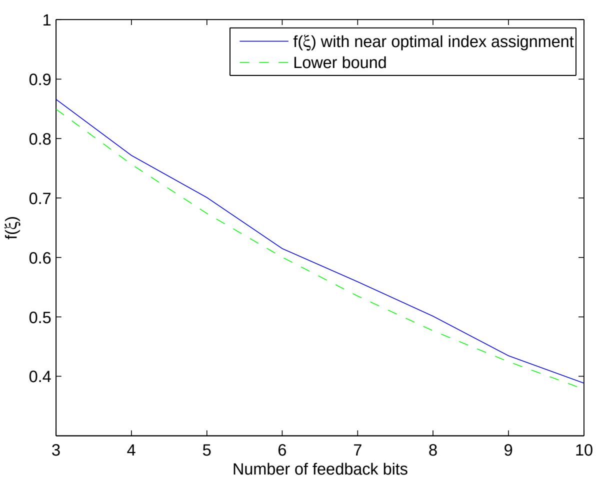


Fig. 3. Accuracy of approximation on $f(\xi) = \mathbb{E}_{\mathcal{I}_k^{(BS)}}(\sin\varphi_{\mathcal{I}_k^{(BS)},i^*})$ in Lemma 4.

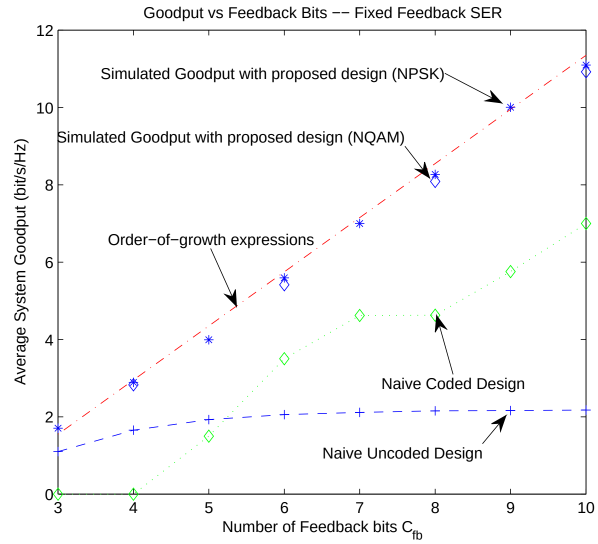


Fig. 4. System goodput versus number of feedback bits. Naive Uncoded Design refers to design assuming the limited feedback channel is noiseless and the feedback bits are uncoded and Naive Coded Design is similar to naive uncoded design except the feedback bits are protected by Hamming code. The SER on feedback channel is fixed at 0.2. The dotted curve of naive coded design is not smooth because of the overhead in the parity bits of Hamming code $(k, 2^k)$ used to protect the CSIT feedback.

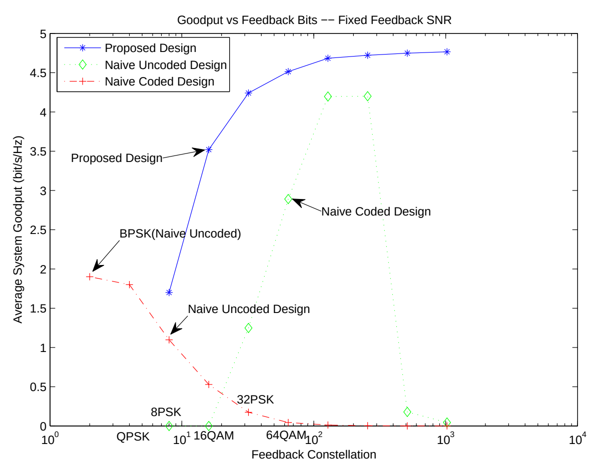


Fig. 5. Average system goodput versus Limited Feedback constellation Level. Naive Uncoded Design refers to the limited feedback design (assuming noiseless feedback) and the feedback bits are uncoded and Naive Coded Design is similar to naive uncoded design except the feedback bits are protected by Hamming code. In all the schemes, the feedback SNR(10dB) and the number of feedback symbols are fixed. The dotted curve of naive coded design drops dramatically when the number of feedback bits gets large because Hamming code only protects nearest neighbor errors.

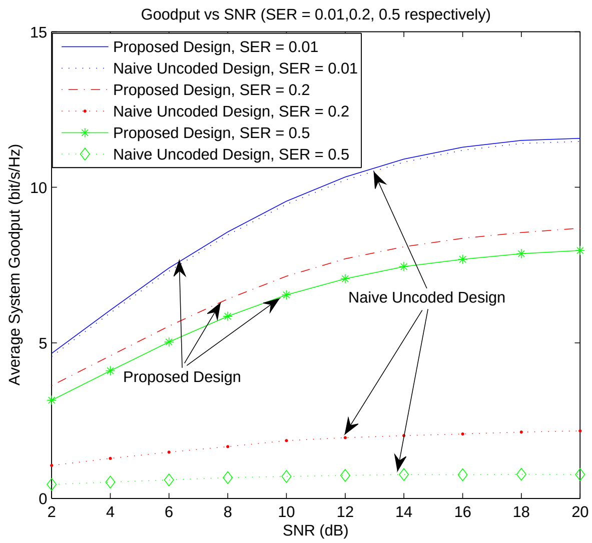


Fig. 6. System goodput versus Forward SNR at different level of feedback SER. Number of feedback bits $C_{fb} = 8$. With proposed design, the system goodput decreases much slower with the increasing of feedback SER.

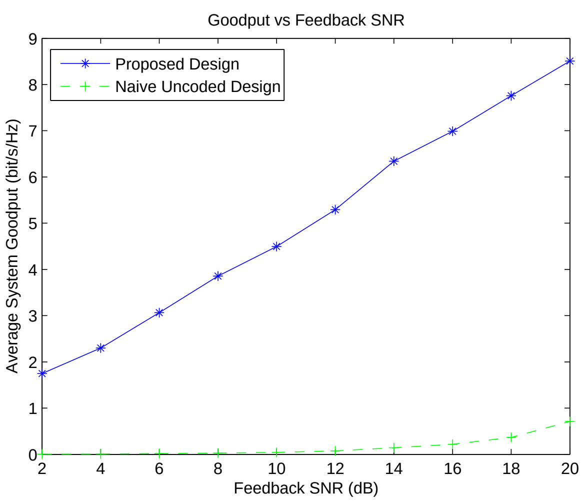
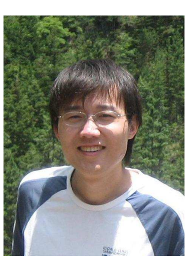


Fig. 7. System goodput versus feedback SNR. Number of feedback bits $C_{fb} = 6$ and forward transmission SNR is 20 dB. The performance of the naive design is very bad since the N_n precoders that may be adopted when feedback error occur are randomly selected and the transmission rate is determined by the worst precoder.



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